Journal of Interdisciplinary and Multidisciplinary Research (JIMR)
E-ISSN:1936-6264| Impact Factor: 8.886| **Scopus Index upto 2016**
Vol. 21 Issue 09, Sep- 2026
Available online at: https://livejimrjournal.in/
(An open access scholarly, peer-reviewed, interdisciplinary, monthly, and fully refereed journal.)

A Comparative Study of Some Small Sample Tests and Large Sample Tests

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Abstract

In this paper, statistical hypothesis testing is an important method for drawing conclusions about populations from sample data. This paper presents a comparative study of selected small-sample and large-sample tests. The study explains their assumptions, applications, test statistics, advantages, disadvantages and differences. The comparison shows that large-sample tests generally rely on normal or asymptotic approximations, whereas small-sample tests often require stronger distributional assumptions and use exact or distribution-specific sampling distributions. The paper concludes that sample size alone should not determine the choice of a statistical test; the research design, independence of observations and distributional assumptions must also be considered.

Keywords: hypothesis testing, small-sample tests, large-sample tests, z-test, t-test, chi-square test, F-test, statistical inference.

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1. Introduction

Statistical inference is based on deciding about the characteristics of the population on the basis of sample study. It also provides a systematic framework for making decisions about a population using information obtained from a sample. In many research situations, it is impractical to collect information from every member of a population. Researchers therefore select a sample and use statistical methods to determine whether the observed results provide sufficient evidence to support or reject a particular hypothesis. A statistical test compares an observed sample result with what would be expected under a specified null hypothesis. The resulting test statistic is evaluated using an appropriate probability distribution to determine whether the evidence is sufficiently strong to reject the null hypothesis.

Statistical tests are commonly classified according to sample size as large-sample tests and small-sample tests. Large-sample procedures frequently use the standard normal distribution or asymptotic approximations. Small-sample procedures, in contrast, often use distributions such as Student's *t*, chi-square, and *F*, particularly when population variance is unknown.

The distinction between small and large-sample tests is useful for understanding statistical methodology, although modern statistical practice recognizes that there is no universal sample-size boundary that automatically determines which test should be used. A test should be selected according to the data, assumptions, study design, and parameter being investigated.

2. Hypothesis Testing

A statistical hypothesis is a statement concerning one or more population parameters. Hypothesis testing normally begins with two competing statements:

2.1 Null Hypothesis (H_0): A statement of no effect, no difference, in which decision maker should adopt a null or neutral attitude regarding the outcome of the test

2.2 Alternative Hypothesis (H_0): Any hypothesis which contradicts the null hypothesis H_0 is called an Alternative Hypothesis and is denoted by the symbol H_1.

A researcher may calculate a test statistic from the sample and determines its probability under the null hypothesis. This probability is commonly represented by the p-value.

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If the p-value is less than or equal to the predetermined significance level α, the null hypothesis is rejected. Otherwise, there is insufficient evidence to reject the null hypothesis.
It is important to distinguish between rejecting H_0 and proving H_0 to be true. Failure to reject the null hypothesis simply means that the available evidence is not sufficiently strong to reject it.

3. LARGE-SAMPLE TEST

Large-sample tests are statistical procedures generally based on normal or asymptotic approximations. Traditionally, a sample size of 30 or more has sometimes been used as a practical rule for identifying a large sample. However, this rule is not universal. The adequacy of a large-sample approximation depends on factors such as the population distribution, variance, sampling design, and parameter being estimated.

3.1 Z- TEST FOR SIGNIFICANCE OF POPULATION MEAN.

For large samples, the standard normal variate corresponding $\bar{x}$ to is:

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Under the null hypothesis H_0, that the sample has been drawn from a population with mean μ and variance σ^2, i.e., there is no significant difference between the sample mean ($\bar{x}$) and population mean (μ), the test statistic is:

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If the population standard deviation. σ is unknown then we use its estimate provided by the sample variance given by $\hat{\sigma}^2 = s^2$ or $\hat{\sigma} = s$; for large samples.

3.2 Z-TEST, SIGNIFICANCE FOR 'DIFFERENCE OF MEANS'

Under the null hypothesis, H_0: $\mu_1 = \mu_2$ i.e., there is no significant difference between the population means,

Thus under H_0: $\mu_1 = \mu_2$, the test statistic becomes (for large samples),

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$, i.e., if the samples have been drawn from the populations with common S.D. then under H_0: $\mu_1 = \mu_2$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

This test is frequently encountered in large sample comparisons between two independent groups.

3.3 Z-TEST FOR POPULATION PROPORTION

If X is the number of individuals (units) possessing the given attribute in n independent trials with constant probability P of success for each trial, then

p= observed sample proportion= x/n

E (X) = nP and V (X)= nPQ,

where Q = 1- P. is the probability of failure. For large samples, the binomial distribution tends to normal distribution.

Hence for large n, X~ N (nP, nPQ), , the standard normal variate corresponding to the statistic p is

$$Z = \frac{p - E[p]}{S.E(p)} = \frac{p - P}{\sqrt{PQ/n}} \sim N(0,1)$$

If we have a sampling from finite population of size N S.E of p is given by

$$\sqrt{\left(\frac{N-n}{N-1}\right)\frac{PQ}{n}}$$

Probable limits for observed proportion of success is given by

$$E(p) \pm 3\, S.E(p) = P \pm 3\sqrt{PQ/n}$$

if P is not known than we take p (sample proportion) as the estimate of P and probable limits for observed proportion of success are

$$p \pm 3\sqrt{pq/n}$$

For large samples (n > 30), the sampling distributions of many statistics are approximately normal distribution.

3.4 Z- TEST, SIGNIFICANCE FOR DIFFERENCE OF PROPORTIONS.

Under the null hypothesis, $\mathbf{H_0 : P_1{=}P_2}$. i.e., there is no significant difference between the sample proportions, we have

$$E(p_1 - p_2) = E(p_1) - E(p_2) = P_1 - P_2 = 0$$

Also $V(p_1 - p_2) = V(p_1) + V(p_2)$

the covariance term Cov(p_1, p_2) vanishes, since sample proportions are independent.

$$\Rightarrow \qquad V(p_1 - p_2) = \frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2} = PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

Under H_0: $P_1 = P_2 = P$ and $Q_1 = Q_2 = Q$

Hence, under H_0 $P_1 = P_2$, the test statistic for the difference of proportions becomes

$$Z = \frac{(p_1 - p_2)}{\sqrt{PQ(1/n_1 + 1/n_2)}} \sim N(0,1)$$

For comparing two independent proportions, a similar z statistic can be constructed using the pooled estimate of the common proportion under the null hypothesis.

4. SMALL-SAMPLE TESTS

Small-sample tests are particularly useful when the sample size is limited and population parameters such as the variance are unknown. Under classical assumptions, several of these tests require the population to be normally distributed.

4.1 Student's t-Test

The Student's *t*-test is one of the most important small-sample procedures. It is commonly used for testing hypotheses concerning population means when the population standard deviation is unknown.

For a one-sample test:The Student's t- statistic is given by

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

where $\bar{x} = \frac{1}{n}\sum_{i-1}^{n} x_i$ and $s^2 = \frac{1}{n-1}\sum_{i-1}^{n}(x_i - \bar{x})^2$ is an unbiased estimate of population variance σ^2, where s is the sample standard deviation.

Under the assumption of independent observations from a normal population, the statistic follows a *t* distribution with: d.f =n−1 degrees of freedom.

4.2 APPLICATIONS:

i. To test if the sample mean ($\bar{x}$) differs significantly from the hypothetical value μ of the population mean

ii. To test the significance of the difference between two sample means.

iii. To test the significance of paired t-test.

iv. To test the significance of an observed sample correlation coefficient and sample regression coefficient.

v. To test the significance of observed partial correlation coefficient.

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4.3 Graph of t-distribution. The p.d.f. of t-distribution with n d.f is:

$$f(t) = C\left(1+\frac{t^2}{n}\right)^{-(n+1)/2} \qquad -\infty < t < \infty$$

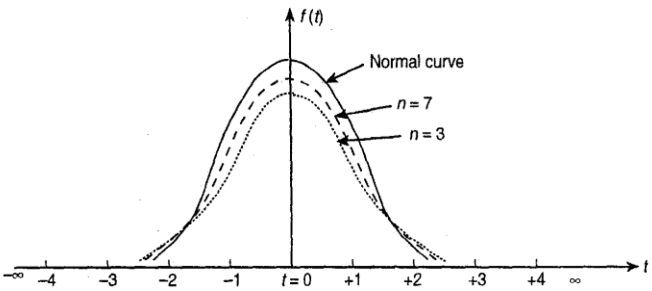
Where $c = \dfrac{1}{\sqrt{n}\,B\left(\frac{1}{2},\frac{n}{2}\right)}$

Since f(-t)=f(t), the probability curve is symmetrical about the line t = 0. As t increases, f(t) decreases rapidly and tends to zero as t $\to\infty$, so that t-axis is an asymptote to the curve. We have shown that

$$\mu_2 = \frac{n}{n-2} \qquad n>2 \;; \qquad \beta_2 = \frac{3(n-2)}{(n-4)}, \quad n>4$$

Hence for n> 2, μ_2>1 i.e., the variance of t-distribution is greater than that of standard normal distribution and for n > 4, β_2> 3 and thus t-distribution is more flat on the top than the normal curve. In fact, for small n, we have

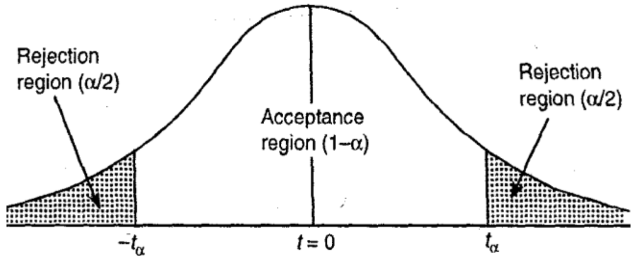
$$P[|\,t\,| \ge t_0] \ge P[|\,Z\,| \ge t_0], \qquad Z \sim N(0,1)$$



i.e., the tails of the t-distribution have a greater probability (area) than the tails of standard normal distribution.

4.4 Critical Values of t. The critical (or significant) values of t at level of significance α and d.f ν for two-tailed test are given by the equation

$$P[|\,t\,| > t_\nu(\alpha)] = \alpha \qquad \Rightarrow \qquad P[|\,t\,| \le t_\nu(\alpha)] = 1-\alpha$$



Since t-distribution is symmetric about t = 0,so we have

$$P[t > t_\nu(\alpha)] + P[t < -t_\nu(\alpha)] = \alpha \qquad \Rightarrow \qquad 2P[t > t_\nu(\alpha)] = \alpha$$

$$\Rightarrow \qquad P[t > t_\nu(\alpha)] = \alpha/2$$

Therefore $\quad P[t > t_\nu(2\alpha)] = \alpha$

$t_\nu(2\alpha)$ (from the Tables) gives the significant value of t for a single-tail test [Right-tail or Left-tail-since the distribution is symmetrical], at level of significance α and ν d.f. Hence the significant values of t at level of significance 'α'for a single-tailed test can be obtained from those of two-tailed test by looking the values at level of significance 2α .

5. Chi-Square Test

The chi-square distribution is used in several statistical procedures The Square of a standard normal variate is known as a chi-square variate with 1 degree of freedom.

Thus if $X \sim N(\mu, \sigma^2) \quad then\ Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$ and

$$Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2 \text{ is a chi-square variate with 1 d.f.}$$

In general if Xi, (i = 1, 2, ..., n) are n independent normal variates with means μ_i and variances a σ_i^2 (i = 1, 2, ..., n), then

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{X-\mu}{\sigma}\right)^2 \text{ is a chi-square variate with n d.f.}$$

5.1 APPLICATIONS

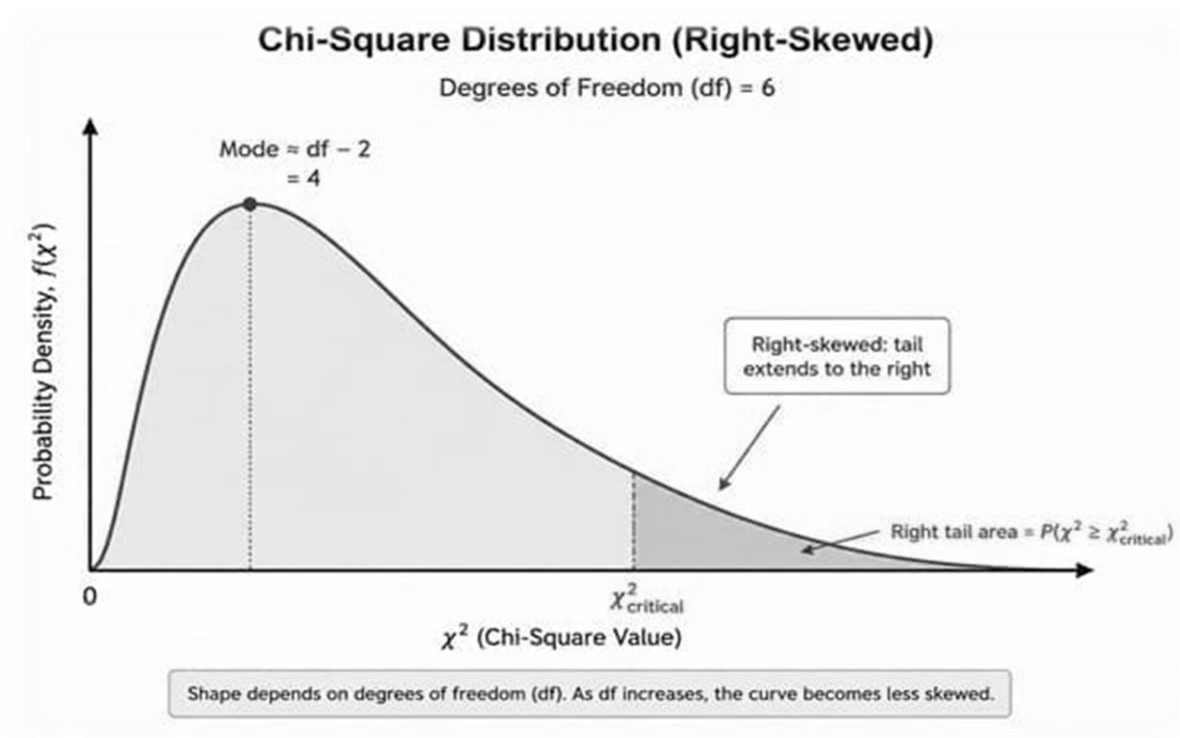
i. χ^2 test for Inferences About a Population Variance

ii. χ^2 test for Goodness of Fit Test.

iii. χ^2 Test of Independence of Attributes

5.2 Shape of Chi square distribution

The chi-squared distribution (chi-square or X^2 distribution) with degrees of freedom, k is the distribution of a sum of the squares of k independent standard normal random variables. It is one of the most widely used probability distributions in statistics. It is a special case of the gamma distribution. This single mathematical fact explains nearly everything distinctive about its shape.



The chi-square distribution is technically a special case of the broader **gamma distribution** family, and it shows up as a building block inside other well-known distributions. The t-distribution and F-distribution, both heavily used elsewhere in Six Sigma hypothesis testing, are defined in part using the chi-square distribution, which makes understanding its shape useful well beyond chi-square tests themselves.

6 F-Test

The F-test is widely used for comparing two population variances under the assumption of independent samples from normal populations.

F-distribution: If X and Y are chi-square variates with ν_1 and ν_2 degrees of freedom respectively, then F-statistic is defined by

$$F = \frac{X/\nu_1}{Y/\nu_2}$$

Hence, F is defined as the ratio of two independent chi-square variates divided by their respective degrees of freedom and it follows Snedecor's F- distribution with (ν_1, ν_2) d.f. denoted by $F \sim F(\nu_1, \nu_2)$ with probability function given by:

$$f(F) = \frac{\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \cdot \frac{F^{\frac{\nu_1}{2}-1}}{\left(1+\frac{\nu_1}{\nu_2}F\right)^{\frac{\nu_1}{2}+\frac{\nu_2}{2}}} \qquad 0 \le F < \infty$$

6.1 APPLICATIONS

i. F-test for Equality of Two Population Variances.

ii. F-test for Testing the Significance of an Observed Multiple Correlation Coefficient:

iii. F-test for Testing the Significance of an Observed Sample Correlation Ratio η_{YX}.

iv. F-test for Testing the Linearity of Regression:

v. F-test for Equality of Several Means

6.2 Shape of F distribution

As we can see in the below given figure, the F distribution has a single mode. The specific shape of an F distribution depends on the number of degrees of freedom in both the numerator and the denominator of the F ratio. But, in general, the F distribution is skewed to the right and tends to become more symmetrical as the numbers of degrees of freedom in the numerator and denominator increase.

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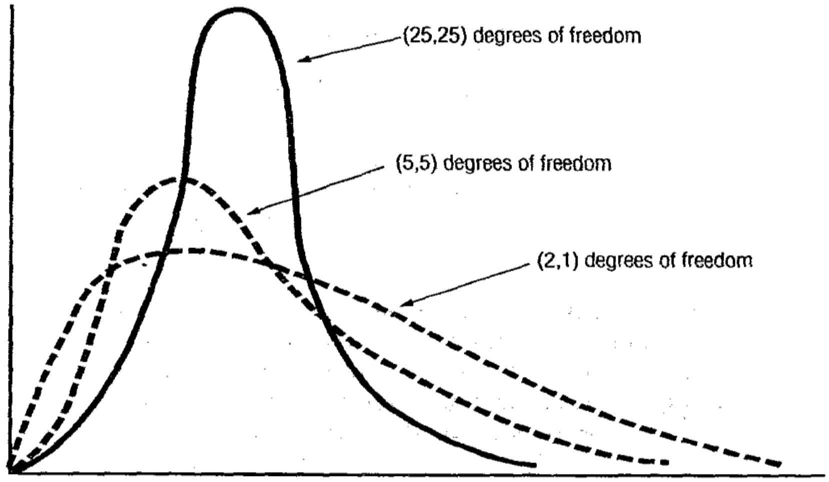

Here we see that the probability p(F) increases steadily at first until it reaches (corresponding to the modal value which is less than 1) and then slowly so as to become tangential at $F=\infty$, i.e., F-axis is an asymptote right tail.

The *F* distribution also forms the basis of analysis of variance (ANOVA), where it is used to test whether several population means can reasonably be considered equal.

7. Comparative Study of Small Sample tests and Large-Sample Tests

Name of Test	**Application**	**Distribution**	**Assumptions**
Z-test	Mean	Standard normal: mean =0, variance=1	Sampling, distribution and known variance
One-sample t-test	Mean of a single group of numbers is different from specific, known or target number	Student's t	Observations must be independent and the data must be approximately normal
Two-sample t-test	Difference between two means	Student's t	Continuous data, random sampling, independent observations and a

			normal distribution for data.
Paired t-test	Mean difference in paired observations	Student's t	Independent pairs; normally distributed
Chi-square test	Variance or categorical data	Chi-square	Categorical variables with independent and mutually exclusive observations, adequate expected frequency.
F-test	Comparison of variances; ANOVA	F distribution	Independence, Normality, Random sampling and Homogeneity of variance.

8. Assumptions of Statistical Tests

Correct statistical inference depends heavily on assumptions. Common assumptions include:

8.1 Independence

Individual observations in the sample must be independent of one another. Violations of independence can lead to incorrect standard errors and misleading conclusions. The selection or value of one data point cannot influence another.

8.2 Normality

The population from which the sample is drawn must follow a normal probability distribution. For small samples, severe departures from normality can affect the validity of these procedures. This is crucial for small samples since skewness or outliers will heavily distort test results.

8.3 Homogeneity of Variance

When comparing two independent small samples, the population variances should be roughly equal, unless using a separate-variance adjustment. When this assumption is doubtful, **Welch's t-test** is often preferable because it does not require equal variances.

8.4 Adequate Expected Frequencies

For Pearson's chi-square test of independence, expected cell frequencies should generally be sufficiently large for the chi-square approximation to be reliable. When expected frequencies are very small, an exact method such as Fisher's exact test may be more appropriate for certain contingency tables.

8.5 Random Sampling

The data must be collected using a simple random sample to ensure it is representative of the target population. A large sample cannot compensate for serious selection bias or a fundamentally inappropriate sampling procedure.

8.6 Linearity

The relationship between your independent and dependent variables should be a straight-line pattern, vital for regression

Statistical tests rely on specific rules about your data called assumptions. If your data breaks these rules, your test results might be wrong.

9. Assumptions for Large Data Samples ($n \geq 30$)

i). **Large Sample Size:** The sample size (n) must be large, usually greater than 30. This size ensures the sampling distribution of the statistic is approximately normal.

ii)**. Independence:** Individual observations must be independent of one another. The value of one data point must not influence another.

iii)**. Random Sampling:** The data must be collected using random sampling of the larger population to prevent systematic bias and ensure representativeness. .

iv)**. Approximate Normality:** While the Central Limit Theorem minimizes the need for the underlying population to be strictly normal when n is large, extreme skewness can still violate the approximation for smaller increments near 30.

v)**. Homogeneity of Variance :** For a two-sample large test (Z-test for two means), traditional pooled variance assumes equal variances. If variances are unequal in large samples, we can use **Welch's adaptation** or unpooled standard error formulas, which do not require strict homogeneity of variance.

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10. Assumptions for Small Data Samples (n < 30)

i. **Normality:** The parent population from which the sample is drawn must follow a normal distribution. This is critical for tests like the student's t test. If the data is skewed, standard parametric tests will fail.

ii. **Independence:** Just like large samples, every data point must be totally independent. The sample observations must be collected independently of one another, meaning one selection does not influence another

iii. **Homogeneity of Variance:**. For tests comparing multiple groups like an independent two-sample t-test or ANOVA, the variances within each group should be approximately equal.

iv. **Random Sampling:** The data must be acquired through simple random sampling to ensure it is representative of the study population.

11. Advantages and Disadvantages of Large-Sample Tests

11.1 Advantages

i) **Lower Sampling Error:** A bigger group reduces random errors, making your final numbers closer to the real population values.

ii) **Higher Statistical Power:** It helps you find true effects or differences even when those effects are small.

iii) **Better Generalization:** Results from a large group apply more easily and accurately to the whole population.

iv) **Normality Assumption:** With the help of Central Limit Theorem, large samples let you use normal distribution rules even if the original data is not normal.

11.2 Disadvantages:

i) **High Cost:** Collecting data from many people or items takes a lot of money.

ii) **Time-Consuming:** Gathering and cleaning a massive amount of data requires extra time and effort.

iii) **Logistical Limits:** Managing huge datasets can overload your software, storage, and team.

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iv) **Over-Significance Risk:** Extremely large samples can flag tiny, unimportant differences as "statistically significant" when they do not matter in the real world.

12. Advantages and Disadvantages of Small-Sample Tests

12.1 Advantages :

i) **Economical**

It requires fewer participants to reduces operational expenses and saves financial resources.

ii) **Expeditious**

Speeds up data collection, to accelerates data entry and fast-tracks immediate analysis.

iii) **Feasibility**

Practical for rare diseases, ideal for crash testing and useful for unique cases.

iv) **Virtuous outcomes**

Minimizes human subject risks, animal testing numbers and limits exposure to hazards.

v) **Simpler Management**

Easier to monitor data and high-quality data to control.

12.2 Disadvantages

i) **Underpowered study**

High risk of Type II errors, may miss real effects and fails to detect small changes.

ii) **Sensitivity to Outliers**

One extreme value distorts results, Skews the sample mean heavily and leads to misleading conclusions.

iii) **Strict Assumptions**

Requires normal distribution data, needs variance homogeneity and incorrect assumptions invalidate results.

iv) **Poor applicability**

Does not represent diverse populations, high risk of selection bias and low external validity.

v) **Wide Confidence Intervals**

Creates high statistical uncertainty, limits precise population estimates to reduces overall test reliability..

Therefore, a sound statistical analysis requires not only knowledge of formulas but also an understanding of when and why a particular test should be applied.

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