
PRINCIPLES AND PHILOSOPHY OF MATHEMATICS- CONCEPT TO CHOREOGRAPHY OF LĪLĀVĀTI

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ABSTRACT:

Indian Drama and dance are governed by the most important element called Rasa. The primary objective of any dance drama production is to bring about Rasa Nishpatti in the Rasika, as Bharata describes in Natyashastra. While Bhava is the cause, present in the dancer, Rasa is the effect induced in the audience. The challenge on hand is the Implementation of a subject like Mathematics and a book of mathematical problems called Līlāvāti into a dance drama production or in very sophisticated terms, bring about the marriage of Art and Science and convey the philosophy clandestinely embedded in Mathematics to the non-technical audience and help them experience the bliss and joy which otherwise could never have been imagined in a mundane mathematical problem.

Līlāvāti by Acharya Bhāskarāchārya is a compilation of mathematical problems encapsulating concepts of Mathematics such as 0, place value, fundamental mathematical operations, fractions, suppositions, permutation and combinations, geometry, trigonometry, series and infinitesimals, to mention a few. A deep study of this poetic and mathematical marvel revealed the philosophy embedded in the interestingly narrated problems. This when envisioned by a dancer, manifested into a production inspired by the concept of value of 0 to infinity. The concept of factorial, manifested into the Vishwa Roopa or Anantha, highlighting the very purpose of one's birth in this world from the point of Agnāna(not realized) to Chetana(realization). Problems were chosen based on the danceability of either the mathematical concept or the application of the concept and woven into a narrative, making the audience traverse through the production without putting them into any type of mathematical

jeopardy. The philosophical background of Bhāskarācharya, the affection of a father towards his daughter and poetic brilliance of acharya are the pillars of the conceptualization, original music composition and choreography. The poetic and imagination of Bhāskarācharya's approach towards Mathematical problems, paved way to establish the sthayi, and the Rasa notably Sringara, along with Veera, Raudra, Shoka and Adbhuta are extracted through the right selection of problems.

The biggest challenge while treating subjects like mathematics, especially a treatise like Līlāvāti, as a dance drama production is:

- Clear understanding of the subject, concepts under consideration.
- A feasibility study about the problems in the book as to whether the concept or its application is danceable.
- Manifestation of a good story line keeping in mind the Satva that the researcher plans to portray. The problems that can envelop well into the story should be chosen.
- A very clear picture of the outcome should be in the researcher's mind, so that production can be driven home in the same direction.
- The various elements of the production like a beginning, process, climax, ending should be integral to the production.
- Careful selection of problems relevant to the story line.

The collective and multidisciplinary nature of a dance production as Līlāvāti keenly allows cross-disciplinary questioning. The research methods used can be performative, action-oriented and experience based. The primary aim being highlighting and analysing the artistic process, as well as experimenting and staging of these experiments. The various aspects like the music, the various forms, the mode of presentation, the narrative aspect the kind of audience being catered to and the theme itself are central to the design of the research. But most important of all is a dialogue with scholarly fields such as philosophy, mathematics, theatre, and music.

Analytical Research formed the beginning stage of the research which involved understanding every aspect of the topic, right from the definition of Mathematics, its origin, its classification. Understanding the treatise of Līlāvāti beginning from the composition of the

book, the concepts addressed, the algorithmic approach with lots of readings supported by the commentaries, the application of the concepts by Bhāskarācharya in the forming of narrative problems manifested into human or animal characters and situations.

A Descriptive Research involved deeper understanding of the problems chosen. A definitive approach as to incorporating the philosophy in each of the mathematical problems. The multidisciplinary proficiency of Bhāskarācharya, compels the researcher to delve into the book and deconstruct it keeping the philosophical element in mind. His proficiency in many fields strongly influences the multi-disciplinary alignment of the problems, with respect to the poetic meter or the musicality that may be incorporated according to the Chandas mentioned.

The concept of drama is driven heavily by the eleven aspects defined by Bharatha in the Natyashastra. Primal among them is Rasa and Bhava, which may be described as the two wheels of an aesthetic performance. Rasa is a unique experience in Indian drama. What is Rasa? It is slightly unfair to try and define Rasa in a superficial manner. It is rather an individual experience internalized by one's understanding and intellectual capability of the Rasika. Locating the Sthayi bhava in a character and in totality of a production earmarks the Rasa Nishpatti in the audience, when driven home using suitable abhinaya. Indian drama productions heavily depend on mythological stories and puranas for their story line, as the stories are well crafted in terms of producing Rasa. With a very clear understanding of all these eleven aspects of Natyashastra it is not wrong to say that it is possible to bring about a drama production based on a subject which many may be written off as Rasa Viheena.

Key words: Lilavati, Bhaskaracharya, Philosophy of mathematics, Aesthetics in mathematics, Khandameru, Bhillama Bhupati, Kharahara, Indian mathematics

Introduction

Līlavāti and Bhāskarācharya:

The ancient Indian Mathematics finds its roots in the Vedas, the holiest oral texts of Hinduism. Mathematics finds its place in 3 Vedangas which are Jyotisa (astronomy), Kalpa (rules of rituals) and Chandas (Science of prosody).

Bhāskarācharya was a multifaceted scholar born in Maharashtra in the year 1114 C.E. At a young age of 36, he wrote Siddhantasiromani, a compendium of various subjects like Arithmetic(Līlavāti), Algebra (Bijaganitam), spherical trigonometry (Goladhyaya) and Planetary mathematics (Grahaganitham). He was proficient in Panini's grammar, The four Vedas, many branches of medicine, five branches of mathematics, poetry, literature and belonged to two schools of philosophy among many more qualities.

Bhāskarācharya is considered as one of the pillars of Indian mathematics and astronomy alongside Varahamihira, Brahmagupta, Aryabhata. Bhāskarācharya went on to become the head of the astronomical observatory in Ujjain where his predecessors had also done some significant works.

The book of interest is Līlavāti by Bhāskarācharya. Līlavāti is a comprehensive collection of mathematical problems related to various branches of mathematics like arithmetic algebra geometry, mensuration, number theory, permutations, combinations, sequences and series. Līlavāti is one of the most celebrated works in the field of mathematics even today. Although written in the 12th century C.E, it is amazing how the concepts and their applications in the form of sutras or problem statements are interesting to implement even to this day. Hence it found its place until 80 years ago before the Cambridge style of education entered the schools during the pre-independence era. Ancient Indian Mathematics in India hence can be understood to be application oriented¹ and the methods were mostly Karanasutras form (algorithm in nature) which suited the application nature. The striking features of Līlavāti which earmarks it, is its poetically articulated presentation of mathematical problems which tingle once brain cells to brood and probe the problem. This very mode of presentation inculcates the interest of the student and feeds his /her intellect. There are many stories rather legends which go about as to

¹ प्रयोजनमनुद्दिश्य न मन्दोपि प्रवर्तते(*prayojanamanuddisya na mandopi pravartate*)
Without any purpose even the dull witted does not proceed further

why Bhāskarācharya named the book as Līlāvāti. Some scholars opine that Līlāvāti was Bhāskarācharya's daughter, as mentioned in some Persian literature, while some think that Līlāvāti was the name of Bhāskarācharya's teacher's daughter, who Bhaskara was in love with. While a more rational thought is that Līlāvāti is more of a metaphorical reference to the playful spirit of the book itself. This is evident from the problems stated by Bhaskara in the book.

Līlāvāti -The Production

Conceptualization of the storyline:

A subject like mathematics lives in one's mind either as a complex challenging subject or to the other extreme nightmare and never imagined to be a topic of a dance drama production but it is evident after repeated reading that if the fundamentals of drama are followed, the task though challenging is achievable. The topic requires detailed reading and understanding the book and most importantly the author, his life and achievements. The theme of the production is completely inspired by Bhāskarācharya, his life and achievements. The various roles he donned in his lifetime, a father, a mathematician, a remarkable political analyst, an unparalleled teacher and a great philosopher. In the interest of the theme of the production, the story of Līlāvāti as his daughter has been adapted. The word Līlāvāti means playfulness. The title is apt because Bhāskarācharya deals with many kinds of problems and concepts in a poetic and practical and fun way. The story is borrowed from popularly known tales about his daughter Līlāvāti. and his efforts to evade tragedy in her life by trying to manipulate her astronomical calculations. References from commentaries, reveal that Bhāskarācharya was appointed as the head of the Observatory in Ujjain by the then ruler Bhillama Bhupati. The storyline meanders around his skills as a political analyst. It is noteworthy that depending on the storyline, concepts or application-based problems are introduced to highlight the application of mathematics in real time. The concept of "Kharahara" or infinity is introduced with the concept of one divided by zero produces infinity. This concept is a bone of contention in modern day mathematics as it is now divided as undefined or indeterminate. The philosophy being when a human (1) is divided or effected by no worldly pleasures (0) he reaches the stage of realization manifested as Anantha, a form of Vishnu, implemented using the concept of Factorial. Hence the journey begins with a father (attached to his daughter) and culminates in the realization (philosopher, detachment).

Script:

The multifaceted Bhāskarācharya is the driving force of the entire conceptualization of Līlāvāti. Bhāskarācharya was a great Sanskrit grammarian and a poet which is reflected in the way the problems are stated beautifully and poetically. This poetic manifestation of mathematics formed the sahitya of the production where the problem statements itself forms the main sahitya. In addition, this sahitya supported by simple versus crafted specially by a scholar to enhance the story line. The sahitya is further embellished with verses from Bhagavad Gita and beautiful verses from Manasollasa. The simplicity of the sahitya is significant keeping in mind the fact that it should be easily understood. The sahitya is written in Sanskrit to maintain neutrality and a big shoulder is lent by narration in the local language or English. Some verses of Līlāvāti are accompanied by simple sahitya written to breakdown the impeccable proficiency and complexity of language.

Problem selection:

The problem selection involved a strategic study of the entire text. This is required because what holds inside the text is a myriad of problems and to the even more myriad number of solutions to them based on a plethora of fundamentals which mathematically leads to an infinite set of stories and emotions and exploratory and scientific approach which resonates with the ultimate aim of the production are handpicked while doing so the selection is based on:

- Danceability and practical implementation.
- If the concept is not danceable and application of the problem is implementable.

The application of a concept is one of the ground breaking processes as a dancer because, today's system of learning has failed to inculcate the love for mathematics, as it does not deliver to the young mind the application of a concept. Great mathematicians wrote these books with a vision to make maths easy.

Music:

The heart of any dance production is the music and with no prewritten music or literature the creative process climbs a notch higher. The selection of ragas is based on the Sthayi of the scene and the rasa to be produced by the problem. Chandas or prosody, an important concept of the Vedas is incorporated suitably in the rendition. The mathematics of Talas such as Yathis,

Gati Bheda, are interspersed into the talas. The talas ageing carefully selected to express the Rasa suitably. The production brings together various art forms like Kuchipudi, Mohiniyattam, kathak and Bharatanatyam on one platform keeping in mind the integrity and the essence of the theme.

Problems and Choreography:

The task of converting text to visual through dance poses a challenge due to

- Lack of aesthetics in maths.
- Limitation of resources and intellect.
- Lack of understanding of the principles and philosophy of mathematics

It is important to maintain a linear flow of the story line, when integrating so many diverse concepts that are non-linear in form.

Problem 1: Place value²

- Important aspect of place value system in Līlāvāti is that just by knowing 10 digits (0 to 9) and 18 position names, one can write any decimal value. This is possible due to the presence of 0 in our mathematics.
- A front facing dancer depicts decimal values like (0, 10, 100 and so on)
- A back facing dancer depicts decimal values below zero like (1/10, 1/100, 1/1000 and so on)
- The jathis are implemented to a swara, sollukattu jugalbandi on either side.

² एकादशशतसहस्रायुतलक्षप्रयुतकोटयः क्रमशः ।
अर्बुदमब्जं खर्वनिखर्वमहापद्मशङ्कवस्तस्मात् ॥(10)

जलधिश्चान्त्यं मध्यं परार्धमिति दशगुणोत्तराः संज्ञाः ।
सङ्ख्याः स्थानानां व्यवहारार्थं कृताः पूर्वैः ॥(11)
The sutra describing the names for each place value

Problem 2: **Khanda Meru**³

- Khanda Meru is represents Pascal's triangle.
- Any value in pascal's triangle is represented by mathematical notation nCr . As we move down rows, we can observe that values in the last row keep increasing, and the number in the centre has the highest value in the row.
- If a king can position his army in this formation, we can observe that the strength of the army increases as we have more rows added and the king well-guarded at the center. So, an army thus positioned can easily attack enemy and easily conquer them and protect the king.
- The concept was driven using mridangam sollukattus to khandachapu supported by powerful rendition and acoustic instruments.
- Rasa: Veera

Problem 3: **Finding unknown using fractions**⁴

- The application of fractions, and the principle of finding the unknown using the known 'X'.
- This algebraic problem is introduced in the backdrop of a yajna that king performs for the prosperity of his kingdom, in the advice of Bhāskarāchārya. A fraction of unknown quantity of flowers are offered to the deities. The known quantity is 6 flowers offered to guru.

³ एकाद्येकोत्तरा अङ्गा व्यस्ता भाज्याः क्रमस्थितैः ।

परः पूर्वेण सङ्गण्यस्तत्परस्तेन तेन च ॥ (112)

मूषवहनभेदादौ खन्दमेरौ च शिल्पके ।

वैद्यके रसभेदीये तन्नोक्तं विस्तृतेर्भयात् ॥(114)

The sutra describes the algorithm for constructing the Khandameru, famous as the Pascal's Triangle

⁴ अमलकमलराशेस्त्र्यंशपञ्चांशष्टै

स्त्रिनयनहरिसूर्या येन तुर्येण चाऽऽर्या ।

गुरुपदमथ षड्भिः पूजितं शेषपद्मैः

सकलकमलसन्ख्यां क्षिप्रमाख्याहि तस्य ॥ (53 or 54). This sutra is based on fractions and finding the unknown.

- The implementation involved a shloka dedicated to each deity followed by a jathi designed and choreographed in Adi talam, Khanda chapu taala.
- Rasa: Sringara (Bhakti)

Problem 4: Factorial⁵

- Earliest representation of factorial is from the Dasa Bhuja Siva sculpture at Badami. All the 10 hands of Siva depict one different weapon. By re-arranging the weapons in these 10 hands one can get a factorial (10) combination.
- Representing this concept more easily using factorial (4)
- Lord Vishnu has 4 bhujas each having a different weapon (like Shankha, Chakra, Gada, Padmam)
- Factorial (4) = 24.
- Lord Vishnu has 24 different combinations of his weapon arrangement, by just rearranging the weapons in his hands.
- Each arrangement gives rise to 24 names of Vishnu.
- The unique feature is that the verses from Manasollasa describing the 24 forms of Vishnu with the names formed by joining the first alphabet of each weapon and correlating the avatara to one of the 24 names of Vishnu rendered during Aachamana.
- Rasa: Bhakti Sringara, Adbhuta

Problem 5: Infinity

- We show $1+0$, $1-0$ and $1/0$.
- Both $1+0$ and $1-0$ result in unchanged value of 1

⁵ “पाशाङ्कुशाहिडमरुककपालशुलैः खट्वाङ्गशक्तिशरचापयुतैर्भवन्ति।

अन्योऽन्यहस्तकलितैः कति मूर्तिभेदाः शम्भोहरेरिव गदारिसरोजशङ्खैः” (263)

MEANING: कति-How many, मूर्तिभेदाः-different idols, भवन्ति-can be formed, अन्योन्य-by exchanging, हस्तकलितैः-weapons in hands, पाश-noose, अङ्कुश-goat, अहिडमरुक-snake, drum, कपाल-skull, शूलैः-trident, खट्वाङ्ग-staff with skull on top, शक्ति-miscile, शर-arrow, चापयुतैः-with bow, शम्भोः—of Siva, गदा-Mase, अरि-disc, सरोज-lotus, शङ्खैः -and conch, हरेरिव-like those of Hari.

- $1/0 = \text{Infinity}$. If one can subdue his desires to nothing (i.e 0) then one (1) can easily attain the Infinity (i.e Paramatma).
- This is depicted as the infinite form of Vishnu i.e Vishwa Rupam.
- Verses from Bhagavadgita enhance the depiction of the Vishwaroopa
- Rasa: Adbhuta

Future Work:

Is it possible that a mathematician who is a philosopher will be able to separate the two or will he be able to embed or has he embedded the ultimate truth in his mathematics and let the mortal humans solve the problem to unlock the ultimate truth.

An inter-disciplinary approach is fundamental in incorporating such a challenging subject in dance. But as the researcher has said, Bhāskarācharya has beautifully intertwined an abstract way to telling a story in Mathematics, which no other mathematician has accomplished. Many problems still remain to be explored and woven into a beautiful story.

Conclusion:

The general belief that rasa viheena topics cannot be implemented is a myth. A deep study into the ancient mathematical text reveals clearly that an application-based approach like Līlāvātī, as unsolvable a puzzle it seems, on exhaustive study, unlocks itself to the reader through the portrayal of problems using animal and human characters mapped to real life, the feeling of uncomplicatedness and practicality. This very approach of Bhāskarācharya leads to manifestation of each of the problem as a real-life event and every problem seems to unravel an aspect of life and makes one to think in terms of philosophy of life, the realization of oneself, “THE BRAHMAN”.



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