Journal of Interdisciplinary and Multidisciplinary Research (JIMR)
E-ISSN:1936-6264| Impact Factor: 6.886|
Vol. 12 Issue 02, Feb- 2017
Available online at: https://livejimrjournal.in/
(An open access scholarly, peer-reviewed, interdisciplinary, monthly, and fully refereed journal.)

ON EXPECTED WAITING TIME FOR M/M/s/m QUEUE WITHOUT PASSING

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Abstract

We consider a multiserver queueing model with finite input source in which no customer can leave the system until all customers who have arrived chronologically earlier have left the system. The expressions for mean waiting time for easy and hard customers in the steady-state are derived. The graphs between expected waiting time (for hard customer) and traffic intensity have been drawn. The expressions for some particular cases have been derived also.

INTRODUCTION

In this long queue of literature, the work of Washburn (1974) attracted our attention not only because of a relatively new concept of "nopassing" but also from the point of view of practical utility of this concept. He considered the queueing model in which customers arrived in poisson fashion with constant arrival rate and leave the system in the chronological order of their arrival. Sharma et. al. (1983) extended the work of Washburn (1974) to limited waiting space case. Jain (1985) analysed the multiserver queue with no passing in which customers are also discouraged due to a long queue assuming the arrival rate to be dependent upon the number of customers present in the system.

Such type of queueing situations occur at car parking places, a narrowboat lock where the service time consists mostly of paper work and a communication network in which packets must be transmitted in a fixed order. One important example is in practice in servicing of automatic machines. Thus the study of queueing models under the restriction of no passing is of immence importance.

In the present paper, we consider a multiserver queueing model with finite input source with poisson arrival and exponential service time distribution. The outstanding and novel feature of the model investigated in this paper is that there is restriction of 'nopassing" on departing customers which simply means that customers depart from the system in the same chronological order in which they arrive. For the analysis purpose the arriving customers are assumed to be composed of two types of customers i.e. easy customers and hard customers. The customers having zero service are called easy customers otherwise they are named time as hard customers. We have drived expression for expected waiting time. The graphs between expected waiting time (for hard customers) and traffice intensity have been drawn. The expressions for some particular cases have been derived also.

MATHEMATICAL MODEL AND ANALYSIS:

We consider a queueing system consisting of m units in which customers are served by s (<m) servers and leave the system in the chronological order of their arrival. The customers are assumed to arrive in a poisson fashion with arrival rate

$$\lambda_n = \begin{cases} (m-n)\lambda & , \quad n = 0, 1, 2, \ldots\ldots, m-1 \\ 0 & , \quad n \geq m \end{cases}$$

and the service rate

$$\mu_n = \begin{cases} n\,\mu & , \quad n = 0, 1, 2, \ldots\ldots, s-1 \\ s\,\mu & , \quad n \geq s \end{cases}$$

The cumulative distributive function (cdf) of service time is

$F(x) = (1 - p) + p\,[1- \exp(-\mu x)],\ 0 \leq p \leq 1$ and $\mu > 0$.

When p = 1, the service times are exponentially distributed. The traffic stream can be considered as composed of two kinds of customers (i) (1 – p) times total of customers are of easy type who have zero service time, (ii) the remaining are hard customers who have poisson arrival rate λ_P and exponential service time with mean service rate $1/\mu$.

Let N(t) be the number of customers at time t. Define $P_n(t)$ = Probability that there are n customers in the system at time t.

and $$P_n = \lim_{n\to\infty} P_n(t)$$

The steady-state probabilities P_n are given by (Medhi, 1982)

$$P_n = \begin{cases} \binom{m}{n} \rho^n P_0 & , \text{for } n = 0, 1, 2, \ldots\ldots, s-1 \\ \dfrac{m!\ \rho^n \ P_0}{(m-n)! \ \ s! \ \ s^{n-s}}, & \text{for } n = s, s+1, \ldots\ldots., m. \end{cases} \qquad \ldots.(1)$$

Where

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{m}{n} \rho^n + \sum_{n=s}^{m} \frac{m! \ \ \rho^n}{(m-n)! \ s! \ \ s^{n-s}} \right]^{-1}$$

and $\rho = \dfrac{\lambda P}{\mu} < s.$

The expected waiting time for customer A who arrives when there are n customers in the system is

$$E(W) = (1-p) \ E(W_e) + p \ E(W_h),$$

where $E(W_e)$ and $E(W_h)$ are the waiting times of easy and hard customers respectively.

To drive the waiting time for a customer A, let there be n hard customers when A arrives. Now assuming that A itself is hard customer then if $n < s$, A will enter the service immediately, and $W_h = \max(x_1, x_2, \ldots.., x_{n+1})$ …. (2)

where X_i ($i = 1, 2, \ldots.., n + 1$) is the residual service time for the customer being served by i^{th} server. The X_i's are independent random variable with

$$F_{X_i}(x) = 1 - \exp(-\mu x).$$

Therefore,

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$$F_{W_h|N(t)}(x \mid n) = [1 - \exp(-\mu x)]^{n+1} \equiv G_{n+1}(x), \quad n < s \qquad \ldots.(3)$$

If $n \geq s$ then T is the waiting time upto entering in service and Y is the time from initiation of service until leaving time system. So that

$$W_h = T + Y$$

and
$$\left.\begin{aligned} E(T) &= \frac{(n-s+1)}{s\mu} \\ F_Y(x) &= G_{s-1}(x) \end{aligned}\right\} , n \geq s \qquad \ldots. (4)$$

Let M_n be the mean of the distribution $G_n(.)$, so that $M_0 = 0$, $M_1 = 1/\mu$ etc. The expected waiting time for hard and easy customers are

$$E(W_h) = \sum_{n=0}^{s-1} M_{n+1} P_n + \sum_{n=s}^{m} \left[\left(\frac{n-s+1}{s\mu}\right) + M_s\right] P_n \qquad \ldots.(5)$$

and

$$E(W_e) = \sum_{n=0}^{s-1} M_n P_n + \sum_{n=s}^{m} \left[\left(\frac{n-s+1}{s\mu}\right) + M_{s-1}\right] P_n \qquad \ldots.(6)$$

let $M_n = \frac{a_n}{\mu}$, so that a_n is given by

$$a_n = \int_0^{\infty} \left[1 - \{1 - \exp(-x)\}^n\right] dx \qquad \ldots.(7)$$

On substituting the values of a_n from equation (7) in the above equations, then equations (5) and (6) become

$$\mu E(W_h) = a_s + \sum_{n=0}^{s-1} (a_{n+1} - a_s) P_n + \sum_{n=s}^{m} \frac{(n-s+1)}{s} P_n \qquad \ldots. (8)$$

and

$$\mu\, E(W_e) = a_{s-1} + \sum_{n=0}^{s-1} (a_n - a_{s-1})\, P_n + \sum_{n=s}^{m} \frac{(n-s+1)}{s} P_n \qquad \ldots.(9)$$

Using equation (1), then equations (8) and (9) can be written as

$$\mu\, E(W_h) = a_s + \sum_{n=0}^{s-1} (a_{n+1} - a_s) \binom{m}{n} \rho^n\, P_0$$

$$+ \sum_{n=s}^{m} \frac{(n-s+1)}{s} \frac{m!\ \rho^n\ P_0}{(m-n)!\ s!\ s^{n-s}} \qquad \ldots.(10)$$

and

$$\mu\, E(W_e) = a_{s-1} + \sum_{n=0}^{s-1} (a_n - a_{s-1}) \binom{m}{n} \rho^n\, P_0$$

$$+ \sum_{n=s}^{m} \frac{(n-s+1)}{s} \frac{m!\ \rho^n\ P_0}{(m-n)!\ s!\ s^{n-s}} \qquad \ldots.(11)$$

Let D be the difference between expected waiting time for hard and easy customers. Therefore

$$D = \mu \left[E\,(W_h) - E\,(W_e) \right]$$

$$= (a_s - a_{s-1}) + P_0 \sum_{n=0}^{s-1} \left[(a_{n+1} - a_n) - (a_s - a_{s-1}) \right] \binom{m}{n} \rho^n \qquad \ldots.(12)$$

By using equation (7), equation (12) becomes

$$D = s^{-1} + P_0 \sum_{n=0}^{s-1} \left[(n+1)^{-1} - s^{-1} \right] \binom{m}{n} \rho^n \qquad \ldots.(13)$$

SOME PARTICULAR CASES

(i) For s = 1 : In this case equation (10), (11), (12) reduce to

$$\mu\, E\,(W_h) = 1 + P_0 \sum_{n=1}^{m} \frac{n \quad m!}{(m-n)!}\ \rho^n \qquad \text{.... (14)}$$

$$\mu\, E\,(W_e) = P_0 \sum_{n=1}^{m} \frac{n \quad m!}{(m-n)!}\, \rho^n \qquad \text{.... (15)}$$

$$D = 1 \qquad \text{.... (16)}$$

(ii) For s = 2 : Here equation (10), (11) and (12) become

$$\mu\, E\,(W_h) = 2 - P_0 \left[1 - \sum_{n=2}^{m} \frac{(n-1) \quad m!}{2^n \ (m-n)!}\, \rho^n \right] \qquad \text{.... (17)}$$

$$\mu\, E\,(W_e) = 1 - P_0 \left[1 - \sum_{n=2}^{m} \frac{(n-1) \quad m!}{2^n \ (m-n)!}\, \rho^n \right] \qquad \text{.... (18)}$$

and

$$D = \frac{1}{2} + \frac{1}{2} \quad P_0 \qquad \text{.... (19)}$$

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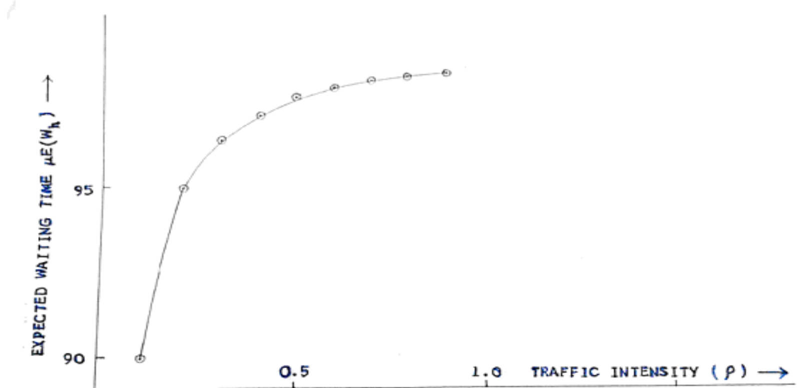
Table : The Expected Waiting Time of Hard Customers

$\mu E(W_h)$ / ρ	s = 1	$\mu E(W_h)$ / ρ	s = 2
0.1	90.000	0.2	46.000
0.2	95.000	0.4	48.500
0.3	96.667	0.6	49.333
0.4	97.500	0.8	49.750
0.5	98.000	1.0	50.000
0.6	98.333	1.2	50.167
0.7	98.571	1.4	50.286
0.8	98.750	1.6	50.375
0.9	98.889	1.8	50.444

Figure – 1 : The Expected Waiting Time For Hard Customers For s = 1



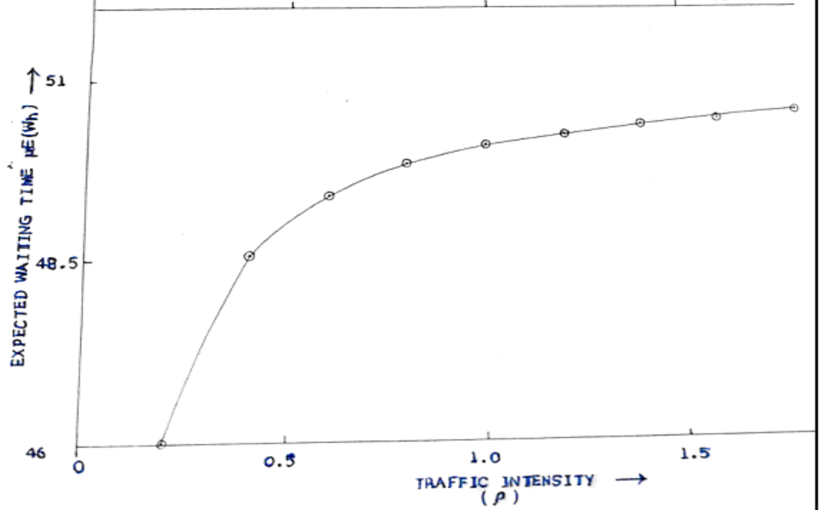


Figure – 2 : The Expected Waiting Time For Hard Customers For s = 2

DISCUSSION:

From the graph, it is clear that waiting time increases sharply till the traffic intansity is 0.4 for s = 1 and ρ = 1.0 for s = 2. And for the values more than ρ = 0.4 for s = 1 and ρ = 1.0 for s = 2, the waiting time does not vary significantly Therefore it can be said that the waiting time stabilizes in this model.

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