Journal of Interdisciplinary and Multidisciplinary Research (JIMR)
E-ISSN:1936-6264| Impact Factor: 6.886|
Vol. 12 Issue 9 September- 2017
Available online at: https://livejimrjournal.in/
(An open access scholarly, peer-reviewed, interdisciplinary, monthly, and fully refereed journal.)

M^x/G/1 DOUBLE ENDED QUEUE VIA DIFFUSION APPROXIMATION

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Abstract

This paper deals with a M^x/G/1 double ended queueing system in which either customers arriving in groups of random size at a service facility with infinite waiting room form the queue for service or idle taxis queue up for customers (passengers). The discrete flow of customers and taxis are approximated by continuous one and the analysis is carried out by using diffusion approximation technique. We provide approximate formulae for mean number of customers and mean number of taxis in the system.

INTRODUCTION

For a M^x/G/1 double ended queueing system, it is difficult to explicit such a system by using generating function method. Although some efforts have been made to investigate double ended queueing systems by using generating method e.g. Sasieni (1961). Double ended queues have varied applications in inventory and production processes. Several researchers have proposed diffusion approximation methods for the single server queueing system e.g. Kimura (1980), with group arrivals.

Fisher (1977) analysed the waiting time at each peripheral station in a loop service system with group arrival. Since stations are assumed to be independent of each other, his results are equivalent to those for M^x/G/1 system. Chaimsiri and moore (1977) discussed the effects of boundary conditions to the accuracy of their diffusion approximate solution for M^x/G/1 system.

Extending their results slightly, Chiamsiri and Leonard (1981) analysed a more general bulk queue denoted by $GI^x/G^y/1$.

In the present study, we consider a double ended queueing system in which customers arrive in groups of random size at a service facility with infinite waiting room. We propose diffusion approximation technique for the present queueing system.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

We consider the double ended queueing model with the following assumptions. The customers are assumed to arrive in a group of random size X according to a poisson process with rate λ ($\lambda > 0$). The group size X is a positive integer-valued identical and independent distributed random variable with a distribution $\{g_n, n = 1, 2, \ldots\}$. The distribution $\{g_n\}$ has the mean υ (≥ 1) and the finite variance σ_g^2.

The arrival of taxis follows poisson distribution with arrival rate μ. Let

$$P_n(t) = \begin{cases} \text{Prob.}\{\text{n customers waiting at time t}\}, & (n \geq 0) \\ \text{Prob.}\{-\text{n taxis waiting at time t}\}, & (n < 0) \end{cases}$$

We assume that $\{X(t) : t \geq 0\}$ is a homogeneous diffusion process in one dimension. We define the probability density function (p.d.f.) of X(t) by

$$P(x, t)\, dx = \text{Prob.}\{x \leq X(t) \leq x + dx\}$$

We shall consider the steady-state solution. Let

$$P_n = \lim_{t\to\infty} P_n(t) \quad \text{and} \quad P(x) = \lim_{t\to\infty} P(x, t)$$

For steady state, P(x) satisfies the Kolmogorov (Fokker Plank) equation

$$\frac{1}{2}\frac{d^2}{dx^2}\{a\,(x)\quad P\,(x)\} - \frac{d}{dx}\ \{b\,(x)\ \ P\,(x)\} = 0 \qquad \text{.... (1)}$$

where a(x) and b(x) are respectively, the infinitesimal mean and variance defined by

$$a(x) = \lim_{\Delta t \to 0} \frac{E\left[X\,(t+\ \Delta t)\ -\ X(t)\ \mid\ X(t) = x\right]}{\Delta t} \qquad \text{.... (2)}$$

and

$$b(x) = \lim_{\Delta t \to 0} \frac{Var\,[X(t+\Delta t) - X(t)\ \mid\ X(t) = x]}{\Delta t} \qquad \text{.... (3)}$$

For $M^x/G/1$ system, we propose

$$a\,(x) = \lambda\upsilon - \mu$$

$$b\,(x) = \lambda\,(\upsilon^2 + \sigma_g^2) + \mu^3\ \sigma_s^2 \qquad \text{.... (4)}$$

where σ_s is the variance of service time.

The diffusion process is confined between two reflecting boundaries at x = -M and X = N. The boundary conditions are

$$\left[\frac{1}{2}\frac{d}{dx}\{a\,(x)\ \ P\,(x)\} - \{b\,(x)\ \ P\,(x)\}\right]_{x=-M \text{ or } N} = 0 \qquad \text{.... (5)}$$

Therefore there is no flow of passengers away from the interval (-M, N) and solution of this equation with boundary conditions is given by

$$P\,(x) = \frac{c}{b\,(x)}\quad \exp\ \int_0^x \frac{a\,(x)}{b\,(x)}\,dx$$

$$= \frac{c}{b\,(x)} \exp(-2Ax) \qquad \text{.... (6)}$$

where $$A = \frac{\mu - \lambda \upsilon}{\lambda\,(\upsilon^2 + \sigma_g^2) + \mu^3\,\sigma_s^2}$$

Using the normalizing condition

$$\int_{-M}^{N} P\,(x)\;dx = 1 \qquad \text{.... (7)}$$

The constant C can be calculated. Then

$$P\,(x) = \frac{2A}{\left(e^{2AM} - e^{-2AN}\right)}\; e^{-2Ax} \qquad \text{.... (8)}$$

On discretizing the P(x), as (Kobayashi, 1974), we obtain the approximate value of P_n,

$$P_n = \int_{n-1}^{n} P\,(x)\;dx$$

$$= \frac{\left(e^{2A} - 1\right)\; e^{-2nA}}{\left(e^{2AM} - e^{-2AN}\right)} \qquad \text{.... (9)}$$

<u>MODIFIED FORMULA:</u>

An approximate formula for the mean number of customers in the system is

$$L_c = \sum_{n=0}^{N} n\,\hat{P}_n$$

$$= \frac{1 - (N+1)\,e^{-2AN} + N\; e^{-2A(N+1)}}{(e^{2AM} - e^{-2AN})\;\;(1 - e^{-2A})} \qquad \text{.... (10)}$$

The mean number of taxis in the system

$$L_T = \sum_{n=-M}^{-1} n\,\hat{P}_n$$

$$= \frac{1-(M+1)\,e^{2AN} + M\; e^{2A(M+1)}}{(e^{2AM} - e^{-2AN})\;(e^{-2A} - 1)} \qquad \text{.... (11)}$$

DISCUSSION

The suggested diffusion approximation technique for $M^x/G/1$ double ended queue is easily applicable to practical queueing situations because the diffusion equations depends only on the mean and variance of arrival processes of customers and taxis. This type of queueing situation are common in inventory and production processes.

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